MAXWELL ON THE LOGIC OF DYNAMICAL EXPLANATION

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In the course of his researches in electromagnetism and the kinetic theory of gases, James Clerk Maxwell gave some thought to the nature of science itself. His observations in this field are of interest today not only because they are his, but because they are still instructive. Maxwell's views are to be found in the many asides with which he enlivened his scientific papers and treatises and in the various articles and reviews which he prepared for more popular consumption. The topics discussed bear on his own contributions to physics; they include the logic of dynamical explanation, the method of physical analogy, and the perennial question of action at a distance versus contiguity. In the present essay, I wish to discuss Maxwell's views on dynamical explanation.[1]

1

What Maxwell called a dynamical explanation is best defined in terms of dynamical analogy which, in turn, is best defined in terms of what he called a physical analogy.

A *physical analogy* is a relation between a branch of one science and a branch of a second science such that both branches possess the same mathematical form, but such that the branch from the first science describes one set of physical phenomena and the branch from the second science describes another set. For example, in the analogy discovered by Lord Kelvin between electrostatics and the theory of heat flow in a solid body, both branches possess the same mathematical form, the lines of electric force corresponding to the lines of heat flow, but the first branch describes the distribution of electricity on conductors and insulators while the second branch describes the motion of heat from the hotter to the colder parts of a body.[2]

What I shall call a *dynamical analogy* is a special kind of physical analogy. In a dynamical analogy, at least one of the two disciplines concerned is a branch of dynamics. The branches of the science of dynamics describe the configuration and motion of what Maxwell called material systems. For example, in the analogy Maxwell himself discovered between electrostatics and the steady motion of an incompressible fluid, both disciplines possess the same mathematical form, the lines of electric force corresponding to the lines of fluid motion or streamlines, but the first discipline describes electrical phenomena while the second discipline describes how a fluid flows from sources to sinks. Another example is Maxwell's analogy between electrostatics and the distribution of tension and pressure in an elastic solid. Here the lines of electric force correspond to the lines along which the tension is exerted and out from which the pressure is directed.

[1] I wish to acknowledge the help received from Professors John C. Cooley and Ernest Nagel in working out the substance of this paper and the suggestions offered by A. C. Danto and Leo Litwak.

[2] For an extended discussion of Maxwell's views on the method of physical analogy, see my paper on this subject (27).

A *dynamical explanation*[3] is also a relation between two disciplines, at least one of which is a branch of dynamics and such that the mathematical form of the one discipline is identical to that of the other. But instead of the sets of properties described by the two disciplines corresponding, the one to the other, the first set of properties is said to be identical to the second set. To put the definition in Maxwell's terms, a dynamical explanation is a dynamical analogy taken literally. Instead of the properties of a material system imitating[4] the properties described by the second discipline, the first set of properties are said to constitute[5] the second set. For example, in the kinetic theory of gases, the collective properties of a large number of material particles constitute the observed properties of gases, for example, the mean kinetic energy of the particles constitutes the temperature of the gas. Maxwell called the branch of dynamics which describes such a material system a physical hypothesis.[6]

To be sure, not all explanations which assert the identity of two sets of properties, described by two different disciplines, are dynamical explanations, for not all such explanations concern branches of dynamics. For example, in the electromagnetic theory of light, electromagnetic waves within a certain range of wave length are said to constitute light waves.

The definition of dynamical explanation is but half the story. Imagination must not outrun nature. If dynamical explanation is to be scientific, then the physical hypothesis must satisfy certain conditions of adequacy. Maxwell, more or less explicitly, recognized two such conditions: the condition of consistent representation and what I shall call the condition of independent evidence.

A physical hypothesis satisfies the condition of *consistent representation* if the hypothesis is proved consistent with the fundamental principles of dynamics which include, for example, Newton's laws of motion and the principle of the conservation of mechanical energy. A new science not only must do justice to the set of physical phenomena it describes, but must not contradict principles established by other sciences.[7] The phrase 'consistent representation' is Maxwell's translation of an expression used in 1845 by Karl Freidrich Gauss in a letter to Wilhelm Eduard Weber.[8] Gauss wrote that he was unable to form a 'construirbar Vorstellung' of how the propagation of electromagnetic disturbances took place in a finite period of time.[9]

[3] See Maxwell (12), 2, 418. He wrote: '. . . when a physical phenomenon can be completely described as a change in the configuration and motion of a material system, the dynamical explanation of that phenomenon is said to be complete.'

[4] See Maxwell (14), 1, 488.

[5] See Maxwell (14), 1, 490.

[6] See Maxwell (13), 1, 155.

[7] See Maxwell (20), §567. He wrote: 'In forming the ideas and words relating to any science, which, like electricity, deals with forces and their effects, we must keep constantly in mind the ideas appropriate to the fundamental science of dynamics, so that we may, during the first development of the science, avoid inconsistency with what is already established. . . .'

[8] See Maxwell (20), §861 and (12), 2, 419.

[9] The letter to Weber is published in (3), 5, 629. Both J. J. Thomson and Sir Joseph Larmor refer to this letter, translating the phrase respectively as 'concrete representation', (26), p. 1, and 'working representation', (5), p. 319.

A physical hypothesis satisfies the condition of *independent evidence* if the properties of the material system it describes can be investigated independently of the science they are designed to explain. Toward the close of his *Electricity and Magnetism*, in reference to an earlier attempt to construct a dynamical explanation of electromagnetic science, Maxwell pointed out that the problem of dynamical explanation always permits of an infinite number of solutions.

> The attempt which I then made to imagine a working model of this mechanism must be taken for no more than it really is, a demonstration that mechanism may be imagined capable of producing a connexion mechanically equivalent to the actual connexion of the parts of the electromagnetic field. The problem of determining the mechanism required to establish a given species of connexion between the motions of the parts of a system always admits of an infinite number of solutions. Of these, some may be more clumsy or more complex than others, but all must satisfy the conditions of mechanism in general.[10]

Unless there is independent evidence to distinguish one explanation from among the infinite number of others, the explanation at best is trivial; it serves only to demonstrate the possibility of dynamical explanation.

I propose first to examine Maxwell's use of dynamical explanation in his researches in electromagnetism, and then to discuss the role this program played in his researches in the kinetic theory of gases.

2

In the first of three memoirs on electricity and magnetism, 'On Faraday's Lines of Force' (13), read 1855, Maxwell demonstrated the dynamical analogy between electrostatics and fluid motion. He also stated that to provide a dynamical explanation of the sciences of electricity and magnetism was the next task for physics. In the second memoir, 'On Physical Lines of Force' (14), published in 1861 and 1862, Maxwell gave his famous field equations and the electromagnetic theory of light and, in what he called the theory of molecular vortices, began the projected dynamical explanation. In this theory, the properties of a system of rotating, spherical cells, strung like beads along the lines of magnetic force, are said to constitute the observed properties of magnetic action. The rotation of the cells causes them to expand laterally and to contract longitudinally which, in turn, causes a tension along the lines of magnetic force and a pressure equal in all directions in a plane at right angles to the lines of force. Further, the properties of a system of tiny particles which move between neighboring vortices when their angular velocites differ, constitute the observed properties of electric action. The theory of molecular vortices to Maxwell's satisfaction met the condition of consistent representation but failed to meet the condition of independent evidence. In the third memoir, 'A Dynamical Theory of the Electromagnetic Field' (9), published in 1864, and in *Electricity and Magnetism*, published in 1873, Maxwell stated that physics for the present must be content with the more modest accomplishment of what he called dynamical theory.

[10] Maxwell (20), §831. See also the account of Lagrange's equations of motion in the chapter 'On the Equations of Motion of a Connected System', (20), esp. §§555–557.

A *dynamical theory* is a dynamical explanation in a less complete form. It seeks that specification of a material system which is at once consistent with the science to be explained and of such generality as to avoid the details which a complete dynamical explanation requires. In a note 'On the Proof of the Equations of Motion of a Connected System', Maxwell discussed the change from dynamical explanation to dynamical theory, using slightly different terms.

> In forming dynamical theories of the physical sciences, it has been a too frequent practice to invent a particular dynamical hypothesis and then by means of the equations of motion to deduce certain results. The agreement of these results with real phenomena has been supposed to furnish a certain amount of evidence in favour of the hypothesis.
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> The true method of physical reasoning is to begin with the phenomena and to deduce the forces from them by a direct application of the equations of motion. The difficulty of doing so has hitherto been that we arrive, at least during the first stages of the investigation, at results which are so indefinite that we have no terms sufficiently general to express them without introducing some notion not strictly deducible from our premisses.
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> It is therefore very desirable that men of science should invent some method of statement by which ideas, precise so far as they go, may be conveyed to the mind, and yet sufficiently general to avoid the introduction of unwarrantable details.[11]

And in a review for *Nature* of Lord Kelvin, then William Thomson, and P. G. Tait's *Natural Philosophy*, Maxwell added:

> But when we have reason to believe that the phenomena which fall under our observation form but a very small part of what is really going on in the system, the question is not—what phenomena will result from the hypothesis that the system is of a certain specified kind? but—what is the most general specification of a material system consistent with the condition that the motions of those parts of the system which we can observe are what we find them to be?[12]

In electromagnetism, the sought for specification of a material system proved to be the equations of motion developed in 1788 by Joseph Louis Lagrange in his *Mécanique Analytique*. Newton's laws of motion and Langrange's equations are equivalent but alternative methods for defining the motion of a material system. In the note on equations of motion and in a chapter on that subject in *Electricity and Magnetism*, Maxwell discussed Lagrange's equations from both the mathematical and physical points of view.[13] From the mathematical point of view, Lagrange's investigations made it possible to reduce Newton's laws of motion, three of which are needed for every particle in a material system, to a number equal to the number of degrees of freedom of that system. From the physical point of view, Lagrange's investigations made it possible to transfer the description of a piece of mechanism from the rigid framework extended through space by Descartes to what Maxwell characterized as the 'independent driving wheels'[14] of the mechanism. Kelvin and Tait called these new coordinates

[11] Maxwell (15), 2, 309.

[12] Maxwell (17), 2, 781. Sir Joseph Larmor referred to Maxwell's new way of looking at physics as the 'agnostic attitude' (5), p. 28.

[13] Maxwell (15), 2, 308 and (20), §§553–554. See also (17), 2, 782–783.

[14] Maxwell (17), 2, 782.

which served to replace those of Descartes, retained coordinates; today they are called generalized coordinates and their time rates of change are called generalized velocities. To apply Lagrange's equations to a material system, it is necessary, first, to determine what are the generalized coordinates and velocities of the system and, second, to determine how the potential and kinetic energies of the system depend upon these quantities. Then it is possible to determine whether the system satisfies the principle of the conservation of mechanical energy. This principle states that the sum of the potential and kinetic energies of a material system remains constant during its motion.

In his review of Kelvin and Tait's work, Maxwell explained the nature of dynamical explanation, why such explanation sometimes must be abandoned, the nature of dynamical theory, and how the latter problem may be solved by using Lagrange's equations. To illustrate, Maxwell described a church belfry with certain special properties.

> In an ordinary belfry, each bell has one rope which comes down through a hole in the floor to the bellringers' room. But suppose that each rope, instead of acting on one bell, contributes to the motion of many pieces of machinery, and that the motion of each piece is determined not by the motion of one rope alone, but by that of several, and suppose, further, that all this machinery is silent and utterly unknown to the men at the ropes, who can only see as far as the holes in the floor above them.[15]

The problem of dynamical explanation is the problem of inferring the nature of the mechanism in the belfry from the observed motion of the ropes. But since there are an infinite number of solutions to this problem and since the mechanism is defined as inaccessible, such explanation is trivial. It cannot satisfy the condition of independent evidence. The problem of dynamical theory is to prove, without recourse to an inaccessible mechanism, that the observed motion of the ropes is consistent with the fundamental principles of dynamics. The solution lies in determining whether Lagrange's equations are applicable to the mechanism in the belfry and whether the sum of the potential and kinetic energies of the mechanism remains constant during its motion. To apply Lagrange's equations, it is first necessary to ascertain the generalized coordinates and velocities of the system. In the problem of the mechanism in the belfry, the generalized coordinates turn out to be the positions of the ropes and the generalized velocities to be the rates of change of these positions. By proper manipulation of the ropes, the bell ringers can determine how to express the potential and kinetic energies of the mechanism as functions of the generalized coordinates and velocities.[16]

[15] Maxwell (17), 2, 783.

[16] Maxwell (17), 2, 783–784. He wrote: 'They can give each rope any position and any velocity, and they can estimate its momentum by stopping all the ropes at once, and feeling what sort of tug each rope gives. If they take the trouble to ascertain how much work they have to do in order to drag the ropes down to a given set of positions, and to express this in terms of these positions, they have found the potential energy of the system in terms of the known co-ordinates. If they then find the tug on any one rope arising from a velocity equal to unity communicated to itself or to any other rope, they can express the kinetic energy in terms of the co-ordinates and velocities.'

Now the history of electromagnetism is the problem of the belfry once again. Ampère's law of attraction and repulsion between current elements and Faraday's law of electromagnetic induction correspond to the observed motion of the ropes. Maxwell's attempt, in his theory of molecular vortices, at the dynamical explanation of these laws corresponds to the attempt to infer the nature of the mechanism in the belfry from the observed motion of the ropes. The more modest task of dynamical theory Maxwell described in *Electricity and Magnetism.*

> What I propose now to do is to examine the consequences of the assumption that the phenomena of the electric current are those of a moving system, the motion being communicated from one part of the system to another by forces, the nature and laws of which we do not yet even attempt to define, because we can eliminate these forces from the equations of motion by the method given by Lagrange for any connected system.[17]

The account of the application of Lagrange's equations to a system of electric circuits is simplified if it is limited to circuits in which the electrical capacity is negligible. For such a system, Maxwell discovered the generalized coordinates to be the set of values necessary to fix the position, shape, and size of each circuit; the generalized velocities to be the time rates of change of these values together with the current strength of each circuit; and the energy of the system to be entirely kinetic in form.[18] With this empirical realization, from Lagrange's equations he obtained the laws of Ampère and Faraday in a slightly generalized form and proved them consistent with the principle of the conservation of mechanical energy.

3

One point in the above discussion of Maxwell's contribution and his interpretation of that contribution requires clarification. Maxwell discovered Lagrange's equations to be the most general specification of a material system consistent with the observed action of electric circuits. His contribution, then, was the demonstration of a dynamical analogy, albeit an analogy in which the corresponding properties are very general. But Maxwell interpeted his contribution as the proof of a dynamical theory, that is, as the proof that the laws of electricity and magnetism describe the observed effects of an intermediate mechanism, albeit the details of the mechanism remain unspecified.

Maxwell never doubted that at bottom the set of all physical phenomena consists in matter in motion as directly observed and in the observed effects of hid-

[17] Maxwell (20), §552. See also §§110–111 in which Maxwell discussed the state of stress in the electrostatic field. He wrote: 'It must be carefully borne in mind that we have made only one step in the theory of the action of the medium. We have supposed it to be in a state of stress, but we have not in any way accounted for this stress, or explained how it is maintained. This step, however, seems to me to be an important one, as it explains, by the action of the consecutive parts of the medium, phenomena which were formerly supposed to be explicable only by direct action at a distance. . . . I have not been able to make the next step, namely, to account by mechanical considerations for these stresses in the dielectric. I therefore leave the theory at this point. . . .'

[18] Maxwell (20), §§568–577.

den, material systems. He granted that the use in electromagnetism of dynamical terms was by courtesy of analogy, but claimed one important exception. In the last of the three memoirs on electricity, 'A Dynamical Theory of the Electromagnetic Field', when discussing his earlier attempt at complete dynamical explanation, Maxwell insisted that energy, irrespective of the various kinds of phenomena by which it became manifest, is always mechanical energy.

> I have on a former occasion attempted to describe a particular kind of motion and a particular kind of strain, so arranged as to account for the phenomena. In the present paper I avoid any hypothesis of this kind; and in using words as electric momentum and electric elasticity in reference to the known phenomena of the induction of currents and the polarization of dielectrics, I wish merely to direct the mind of the reader to mechanical phenomena which will assist him in understanding the electrical ones. All such phrases in the present paper are to be considered as illustrative, not as explanatory.
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> In speaking of the Energy of the field, however, I wish to be understood literally. All energy is the same as mechanical energy, whether it exists in the form of motion or in that of elasticity, or in any other form. The energy in electromagnetic phenomenon is mechanical energy.[19]

And in the very last paragraph of *Electricity and Magnetism*, Maxwell quoted Torricelli's assertation that energy '. . . is a quintessence of so subtile a nature that it cannot be contained in any vessel except the inmost substance of material things.'[20]

The conviction that all energy is mechanical energy is also reflected in Maxwell's interpretation of the electromagnetic theory of light. Instead of taking electromagnetic properties to constitute those of light, he claimed that the properties of a material system constitute both electromagnetic and optical properties. In 'On Physical Lines of Force', the memoir which first gave mathematical expression to the electromagnetic theory of light, Maxwell discussed the evidence for this theory. After noting that the ratio of the electrostatic unit of charge to the electromagnetic unit of current, as calculated from the experiments of Kohlrausch and Weber, had the dimensions and approximate magnitude of the velocity of light in air, as measured by Fizeau, Maxwell concluded: '. . . we can scarcely avoid the inference that *light consists in transverse undulations of the same medium which is the cause of electric and magnetic phenomena.*'[21] Similar comments occur in the later memoir 'A Dynamical Theory of the Electromagne-

[19] Maxwell (9), 1, 563–564. See also (8), 2, 227: 'Thus the words Velocity, Momentum, Force, &c. have acquired certain precise meanings in Elementary Dynamics. They are also employed in the Dynamics of a Connected System in a sense which, though perfectly analogous to the elementary sense, is wider and more general. . . . There are certain electrical phenomena, again, which are connected together by relations of the same form as those which connect dynamical phenomena. To apply to these the phrases of dynamics with proper distinctions and provisional reservations is an example of a metaphor of a bolder kind; but it is a legitimate metaphor if it conveys a true idea of the electrical relations to those who have been already trained in dynamics.'

[20] Maxwell (20), §866. The quotation is from *Lezioni Accademiche* (Firenze, 1715), p. 25.

[21] Maxwell (14), 1, 500.

tic Field',[22] and in the account of the electromagnetic theory in *Electricity and Magnetism.*

> If it should be found that the velocity of propagation of electromagnetic disturbances is the same as the velocity of light, and this not only in air, but in other transparent media, we shall have strong reasons for believing that light is an electromagnetic phenomenon, and the combination of the optical with the electrical evidence will produce a conviction of the reality of the medium similar to that which we obtain, in the case of other kinds of matter, from the combined evidence of the senses.[23]

Indeed, Maxwell not only believed that the properties of a material system constitute both electromagnetic and optical properties, but he believed that someday a knowledge of such things as '. . . whether the electric current is really a current of a material substance, or a double current, or whether its velocity is great or small as measured in feet per second . . .'[24] would make possible the beginning of an acceptable dynamical explanation of electromagnetism.

> A knowledge of these things would amount to at least the beginnings of a complete dynamical theory of electricity, in which we should regard electrical action, not, as in this treatise, as a phenomenon due to an unknown cause, subject only to the general laws of dynamics, but as the result of known motions of known portions of matter, in which not only the total effects and final results, but the whole intermediate mechanism and details of the motion, are taken as the objects of study.[25]

4

I now propose to show that in his researches in the kinetic theory of gases or, as he called it, the dynamical theory of gases, Maxwell again interpreted his results as incomplete and believed that someday physics would remedy this deficiency.

In an article for *Nature* on 'Molecules', Maxwell drew a distinction between the properties of a set of entities investigated individually and the properties of these same entities when investigated collectively. He called the study of the first kind of property the *historical* or *dynamical method* and the study of the second kind of property the *statistical method.*[26] The physical hypothesis of the kinetic theory of gases describes the motion of small material particles called molecules which cannot be directly observed. Maxwell assumed that, while an individual molecule, upon impact with another molecule, changes its velocity, the distribution of molecular velocities over the collection of the molecules remains constant. With this assumption, Maxwell proved the distribution of the molecular velocities to be of the form of the law of errors. The physical hypothesis does not state

[22] Maxwell (9), 1, 580.
[23] Maxwell (20), §781.
[24] Maxwell (20), §574.
[25] Ibid.
[26] Maxwell (10), 2, 374. The distinction between the dynamical and statistical methods is in current use, and has been credited to Maxwell. See, for example, Gibbs (4), p. viii; Cohen (1), p. 139; and Wiener (28), p. 81 footnote.

which molecules have which velocities; it ignores individual histories. In this sense the theory, for Maxwell, was incomplete.

Maxwell's belief that the statistical method was a temporary expedient, necessary only until physics gained a more complete knowledge of the motion of the molecules is apparent from certain other remarks which appear in 'Molecules'. He observed that the statistical method describes 'a new kind of regularity, the regularity of averages', but that this regularity is not 'a perfect knowledge of all the data'; it is just something sufficient 'for all practical purposes'.

> The equations of dynamics completely express the laws of the historical method as applied to matter, but the application of these equations implies a perfect knowledge of all the data. But the smallest portion of matter which we can subject to experiment consists of millions of molecules, not one of which ever becomes individually sensible to us. We cannot, therefore, ascertain the actual motion of any one of these molecules; so that we are obliged to abandon the strict historical method, and to adopt the statistical method of dealing with large groups of molecules.
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> The data of the statistical method as applied to molecular science are the sums of large numbers of molecular quantities. In studying the relations between quantities of this kind, we meet with a new kind of regularity, the regularity of averages, which we can depend upon quite sufficiently for all practical purposes, but which can make no claim to that character of absolute precision which belongs to the laws of abstract dynamics.[27]

The belief that the statistical method is a temporary expedient is also apparent in Maxwell's discussion of a possible contradiction arising in science from the coexistence of two methods of investigation.[28] In *Theory of Heat*, Maxwell invented his famous demon to illustrate the possibility that the properties of a set of entities investigated by the historical method might conflict with the properties of these same entities when investigated by the statistical method. With the aid of a simple mechanical device, the demon, supposed able to observe individual molecules, could upset the second law of thermodynamics in its statistical interpretation.

> . . . we conceive a being whose faculties are so sharpened that he can follow every molecule in its course, such a being, whose attributes are still as essentially finite as our own, would be able to do what is at present impossible to us. For we have seen that the molecules in a vessel full of air at uniform temperature are moving with velocities by no means uniform, though the mean velocity of any great number of them, arbitrarily selected, is almost exactly uniform. Now let us suppose that a vessel is divided into two portions, A and B, by a division in which there is a small hole, and that a being who can see the individual molecules, opens and closes this

[27] Maxwell (10), 2, 374.

[28] Still further evidence for this belief is furnished by Maxwell's assertation that when physics is unable to investigate, because of the smallness of their magnitude, what he called water-shed phenomena, it has to resort to the statistical method. An unstable condition is present in water-shed phenomena; a very small cause can produce a very big effect, like the slight push given a precariously balanced boulder which causes a landslide. Maxwell contrasted water-shed phenomena to phenomena subject to the principle of continuity in which a small variation in cause produces only a small variation in effect. See Maxwell (21) and (18), pp. 13–14.

hole, so as to allow only the swifter molecules to pass from A to B, and only the slower ones to pass from B to A. He will thus, without expenditure of work, raise the temperature of B and lower that of A, in contradiction to the second law of thermodynamics.[29]

5

The inventory of Maxwell's thought has a credit and a debit side. On the credit side, in electromagnetism, Maxwell did limit his researches, after the retreat from complete dynamical explanation, to what today is called the macroscopic domain; while in kinetic theory, the push into the microscopic domain was supported by independent evidence. On the debit side, Maxwell remained convinced that all energy was mechanical energy. However, he must not be charged too much for this conviction. It was less a dogma about the nature of the universe, than a program for unifying physics, creating new theory, and discovering new sets of physical phenomena.

In 'On Action at a Distance', Maxwell observed that some forces in nature seem to act at a distance from separate centers, for example, gravity, while other forces seem to act through an intervening medium, for example, the expanding circles produced when a stone is tossed into a pond. He also observed that the two kinds of forces each served in programs in which physicists attempted to explain away the other kind of force. Maxwell joined the partisans of contiguous action because it seemed to him more 'philosophical', more scientific.[30]

> Why then should we not admit that the familiar mode of communicating motion by pushing and pulling with our hands is the type and exemplification of all action between bodies, even in cases in which we can observe nothing between the bodies which appears to take part in the action?[31]

Dynamical explanation did produce physics from Newton's time well into the 19th century. In Maxwell's researches it advanced both the theory of electricity and the theory of matter. But, as things turned out, these advances helped to establish the limits of the program that produced them.

To close, I wish to mention the comments by Henri Poincaré and, more recently, by Arturo Rosenblueth and Norbert Wiener on the possibility of an

[29] Maxwell (19), pp. 328–329. See also (16), 2, 670. And see also Maxwell's discussion of the use of the statistical method in the social sciences. It parallels his discussion of the use of the statistical method in physics in that again Maxwell saw a possible contradiction arising because of the coexistence of a statistical method and a historical method, (10), 2, 373–374.

[30] Maxwell (11), 2, 312.

[31] Ibid. See also (20), §502. Comparing his program with that of the scientists on the continent, Maxwell wrote: 'The ideas which I have attempted to follow out are those of action through a medium from one portion to the contiguous portion. These ideas were much employed by Faraday, and the development of them in a mathematical form, and the comparison of the results with known facts, have been my aim in several published papers. The comparison, from a philosophical point of view, of the results of two methods so completely opposed in their first principles must lead to valuable data for the study of the conditions of scientific speculation.'

infinite number of solutions to the problem of dynamical explanation.[32] Poincaré's comments are of interest not only because of their clarity, but because more than once he has himself been credited with having discovered this possibility.[33] In the Preface to his *Électricité et Optique*,[34] published in 1901, and in the discussion of Maxwell's researches in the *Foundations of Science*,[35] Poincaré explained how Maxwell proved the action of electric circuits to be consistent with the fundamental principles of dynamics, how the generality of this demonstration made it possible to ignore both the details of a mechanism and the connection between the mechanism and the set of observed phenomena, and how the demonstration thus established the possibility of an infinite number of solutions to the problem of dynamical explanation without constructing a single one of them.

In their article for *Philosophy of Science* on 'The Role of Models in Science', published in 1945, Rosenblueth and Wiener, without mentioning Maxwell, generalized his observation to the observation that any kind of explanation permits of an infinite number of solutions. Instead of a belfry with ropes leading to an inaccessible mechanism, as an illustration, they offered a closed box with inputs and outputs connected to a system of electric circuits hidden within the box. The same relation among the inputs and outputs may be explained by different arrangements of the circuits, different currents, resistances, and so on.

> It is possible to obtain the same output for the same input with different physical structures. If several alternative structures of this sort were inclosed in boxes whose only approach would be through the input and output terminals, it would be impossible to distinguish between these alternatives without resorting to new inputs, or outputs, or both.[36]

To distinguish one explanation from among the infinite number of others, requires independent evidence.

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[32] The possibility of an infinite number of solutions to the problem of dynamical explanation may even become a source of humor as in the cartoons of Rube Goldberg in which any number of mechanisms are introduced, one acting upon the next, in the performance of such simple functions as blowing out a match.

[33] See, for example, Cohen (1), p. 85; Schlick (25), p. 29; Margenau (7), p. 77; Lindsay and Margenau (6), p. 191; and Devereux (2), p. 705.

[34] Poincaré (22), pp. vii–viii.

[35] Poincaré (23), pp. 174–183.

[36] Rosenblueth and Wiener (24), pp. 318–319.

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